NUMERICAL REPRESENTATION OF VARIANTS OF ORALLY TRANSMITTED TUNES

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Man cannot even err at will.
P. Dubois-Reymond

It is a well known fact that orally transmitted (folk) tunes exist in many variants, which differ from each other musically, textually, and even in the language used. Sometimes the affinity of a variant with an ideal model-tune is obvious, sometimes it is hard to recognize, harder to establish. In comparative musicology and folklore the proof of the affinity of a variant or contrafact with a model-tune is a frequent problem; it can rarely be solved with mathematical accuracy. For the oral tradition of a tune is ever-changing, depending upon the place, time, and the singer; and even when it is notated, the musical signs and symbols fix only discrete tones while neglecting the frequent *portamenti di voce* or other ligatures, not to mention the many personal habits in the voice production of the singer or transmitter. The problem of identifying and relating the variants of a model-tune is particularly important, when we compare two or more melodies with different texts, which appear or are claimed to be variants of one and the same model-tune. This variety of texts is quite typical for the traditional chants of the Synagogue. In all such cases the musicologist's task consists in the search for means which, more exactly than the conventional notation and its musical analysis, may demonstrate the existence or non-existence of a relationship between the melodic lines under investigation. The oscillograph and melograph offer rather circumstantial assistance, but their data are too cumbersome to reproduce in a way that truly "represents" the direction, range, interval-structure, etc. of the tune. Such tangible characteristics are especially wanted for melodies which seem to belong to the same family or mode. It may happen that the same few tones that comprise a model-tune recur with different texts and in different languages. When such a tune consists of only a very few tones, their permutation sometimes results in variations, which completely change their notational image. A classic example of such a tune is the globally known three-tone children's song.

As the number of real constituent tones remains constant, the expression $\frac{\text{IQ} \times \text{RT}}{\text{Nn}}$, where IQ stands for the quotient of rising intervals through descending intervals, RT for real (constituent) tones, and Nn represents the number of notes, is a monotonically falling function. In mathematical symbols: $\frac{\lim IQ \times RT}{Nn \to \infty}$ tends to 0. In such cases, when the number of real (constituent) tones is relatively small and constant, while the number of notated tones ever increases, there does not exist a lower bound for the above expression. This is the border, beyond which simple enumeration of tones and interval structures will not yield a meaningful result.

I

A description of a tune is not complete, unless it marks at least the accented tones (in comparison with the unaccented ones), in order to state rhythm and meter. This can usually be done by fairly conventional symbols. It is much more difficult, for the reasons mentioned above, to give an exact mathematical representation of a variant to be compared with a (hypothetical) model-tune. Perhaps the new discipline of "almost periodic functions" will be of help in the future, if we search for exact mathematical solutions. Forgoing these rather complex methods, we should meanwhile search for numerical (not mathematical) methods of approximation. Any such numerical representation will be rather narrowly conditioned: one may cogently and sensibly compare only such tunes that are based upon an equal or closely related tonal system, e.g. tetrachordal, or diatonic-chromatic, or pentatonic, respectively. This limitation clearly refers also to space and time, i.e. to the tune's age and origin. To compare, say, a negro spiritual song that may have some *valse* elements, with a Viennese waltz of the nineteenth century will not only be meaningless, but also fruitless, and conclusions based upon incidental similarities will be problematic, to say the least. As tempting as it may appear to compare an ancient Greek tune with a modern Hellenic song, be it of rural or urban origin, the results of such a comparison will in most cases appear to be non-commensurable – even if similarities might appear.

The problem with which this article deals, refers only to orally transmitted tunes. Why this restriction? To answer the question, it will be useful to state a few observations concerning oral tradition. If any phenomenon is measured by precision instruments, it will yield data which in every measurement vary from each other, if only in relatively small quantities; e.g. when the quantity and intensity of light of a star is measured ten times under more or less equal circumstances, or when the weight of a certain liquid is measured under equal temperature and pressure, the results will never be fully identical. The mathematical "law of error" deals with these deviations and with the problem in general. It is of so great an importance in natural science, that a celebrated physicist was moved to say: "man cannot err at will." One distinguishes between "elementary" errors such as the imperfection of measuring tools, "incidental" errors, "transmission errors" etc. One may compare a series of observational results with the series of variants of an orally transmited tune - from a mathematical point of view these series of data are more or less equivalent in nature. This equivalence can be demonstrated graphically by the curves that appear, when data of the variants are grouped according to their numerical representation. Yet the mathematical law of error does not help us to represent a single measurement or a single variant of a melody, respectively. Hence, we must content ourselves with much less sophisticated methods of *numerical* (not mathematical) approximation.

Thesis I: from a mathematical point of view the n variants of an orally transmitted melody M relate themselves to each other much as the n' different measurements of a mass or phenomenon M'.

II

In every process of approximation one has to ask oneself: how big is the maximal possible error? The question will receive different answers from fieldworkers of folk music and from pure theoreticians. The first group knows from experience that the same tune sung by a singer today will exhibit some slightly different parameters tomorrow. The experienced fieldworker will, therefore, be inclined to anticipate a certain average deviation in two or more renditions of a tune. He might content himself, for example, with introducing quartertones plus or minus the conventional twelve chromatic semitones that comprise the octave. As our system of temperature makes similar concessions by allowing for only one mathematically "pure" interval (the octave), the fieldworker will be justified by the natural limitations of temperature and the human ear. The theoretician, however, will insist upon exact tonal frequencies based on the Ellis scale. This method is sharp enough to define and identify any melody orally transmitted, but it is cumbersome, almost useless, in the comparison of two variants of a tune, transmitted with different texts, let alone in different languages. One perceives the differing numbers on paper, but, even when quartertones are notated in the transcription, it is rather hard to sing them well, let alone to observe them clearly. Moreover, the "pure" system disregards certain continuities, such as *glissandi, portamenti di voce*, etc.; it can fix only discrete tones in discrete frequencies. A recurrent and typical feature of many folk tunes are certain "pivotal tones" as G. Abraham and C. Sachs already realized more than fifty years ago; they would hardly be perceptible in the numbers of the Ellis scale.

There are many ways to put these characteristic features of a folk tune into a meaningful and tangible relationship with each other. Each of these characteristics represents a *parameter* of the melody in the strict sense of the word: the number of notated (or audible) tones, the number of constituent tones, the intervals descending and ascending; the tones omitted in the scale to which the tune may belong, the numer of different tones (tenores) in a psalmodic melody, the melismatic syllables in comparison with the strictly syllabic phrases, the extra-scalar tones, etc., etc. Each consistently followed method, if applied, will yield n melodies or variants, expressed by a different series of numbers. If these series are ordered and graphically grouped in a diagram, certain curves, well known in statistics, will result, indicating the way in which the variants are "distributed," i.e. related to each other.

III

We shall now describe a simple, almost primitive method of numerical approximation that will be used in the following. It is based upon the variations of certain parametric functions of every tune. The parameters chosen for our examples are those which, without logarithmic computation, will yield an acceptable result. They are: (1) the number of notated and constituent tones (neglecting all ligatures); (2) the quotient of ascending intervals over descending ones; the number of tones that constitute an example. Finer methods describe, especially for longer melodies, the proportion of syllables to tones, an important consideration for melismatic tunes.

The maximal error must certainly not exceed a quarter-tone, which corresponds to 0.5, as the semitone corresponds to 1. The widest deviation *either way* (plus or minus) must not exceed 0.25. The musical relationship of these applications to the variants establishes criteria of similarity or identity. In other words: if we examine three "independent" melodies, and their numerical representations are all of the same magnitude, i.e. they differ by less than 0.25 from each other, these variants are *organically*, not accidentally, related to each other.

Thesis II: a number of variants, the parametrical analysis of which yields results whose differences are less than ± 0.25, are organically linked, similar or congruent to each other.

To exemplify the method chosen, we shall first compare three melodies with each other, which clearly belong to the same family or mode, the so-called *Magen avôt* mode. We selected: (1) *Wa-yevareḵ* (Baer, *Baal Tefilla*, no. 407, p. 102; Fig. 1); (2) *Yigdal* (*Ibid.*, no. 760, p. 169; Fig. 2); and (3) *Rīšônah yavô eliyyahû* (*Makkabi-Liederbuch*, Berlin 1930, p. 61; Fig. 3). Each of these tunes will be quoted in its notation.

Fig. 1. *Wa-yevarek*

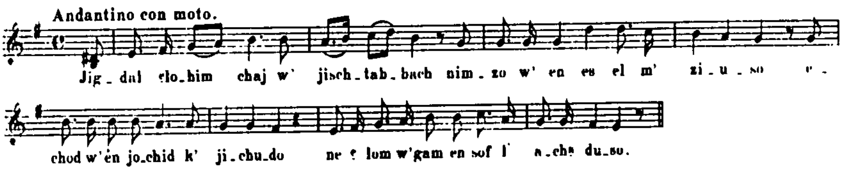

Fig. 2. *Yigdal*

Fig. 3. *Rīšônah yavō eliyyahû*

The following abbreviations are used in the analysis:

IQ = interval-quotient $\left(\frac{\text{ascending intervals}}{\text{descending intervals}}\right)$

RT = number of real tones that appear at least once in the melody.

Nn = number of total notes that comprise the melody

The characteristic parameter (CP) of the melody is the product $\frac{IQ \times RT}{Nn}$

(1) *Wa-yevareḵ*: $IQ = 1;\ RT = 8;\ Nn = 39;$

$$CP = \frac{1 \times 8}{39} = 0.205128$$

(2) *Yigdal:* $IQ = \frac{36}{32} = \frac{9}{8};\ RT = 8;\ Nn = 44;$

$$CP = \frac{9}{8} \times \frac{8}{44} = 0.20454$$

(3) *Rišônah yavô eliyyahû:*

$$IQ = \frac{40}{40} = 1;\ RT = 8;\ Nn = 39;$$

$$CP = \frac{1 \times 8}{39} = 0.205128$$

The difference between 2 and 1 is 0.000588, which amounts to less than one thousandth.

The difference between (1) and (3) = 0. This last result is extremely rare; it points to a common origin of both melodies.

In the subsequent examples only the sources and the results are cited, as the music, the numbers, and their computation would take up too much space. The first group of examples are taken from H. Avenary, *The Ashkenazic Tradition of Biblical Chant Between 1500 and 1900* (Tel Aviv, 1978), pp. 38-9.

A. The accents *zarqā, segôl, munnaḥ, revîᵃa, mahpaḵ, pašṭā, zaqef qaṭon*, are cited in the East-European versions of (a) P. Minkowski, (b) Weisser, and (c) Y. Ne'eman

(a) CP = 0.1624; (b) CP = 0.1944; (c) CP = 0.233.

The differences are:

b - a = 0.0320; c - b = 0.0386; c - a = 0.0706.

B. North-Western Europe: accents *zaqef gadôl, dargā, tevîr, merḵā, ṭippeḥā, etnaḥtā, sôf pasûq*; after (a) J. Reuchlin (1518), (b) Nathan[1] (1823), (c) J. Perlzweig.

(a) CP = 0.17857; (b) CP = 0.2147; (c) CP = 0. 2185.

b - a = 0.0353; c - b = 0.0058.

C. West-Ashkenazi: (a) J. Reuchlin; (b) M. Kohn; (c) S. Sulzer; (d) Naumbourg; (e) Japhet; (f) A. Baer; (g) E. Birnbaum.

Accents 1 - 14: (a) CP = 0.10728; (b) CP = 0.11778; (c) CP = 0.1264; (d) CP = 0.12272; (e) CP = 0.120226; (f) CP = 0.10384; (g) CP = 0.1085.

Maximum (c) 0.1264; minimum (f) 0.10384.

c - f = 0.02256.

1 From his *Musurgia vocalis* (London, 1823).

(according to Ch. Vinaver, *Anthology of Jewish Music*): (a) CP = 0.0963; (b) CP = 0.1172; (c) CP = 0.0953.
b − a = 0.0209; b − c = 0.02188

In order to demonstrate the applicability of our method to medieval European folksong, we shall now test it using some examples of folk songs, following W. Danckert's fine volume *Das europaeische Volkslied*[2] (Fig. 4):

I. Two variants of "Christ ist erstanden": (a) general German version; (b) version of the enclave of Gottschee, (Yugoslavia).
(a) CP = 0.175; (b) CP = 0.16269
a − b = 0.0123

Fig. 4. "Christ ist Erstanden"

2 2nd edition (Bonn, 1970), p. 66.

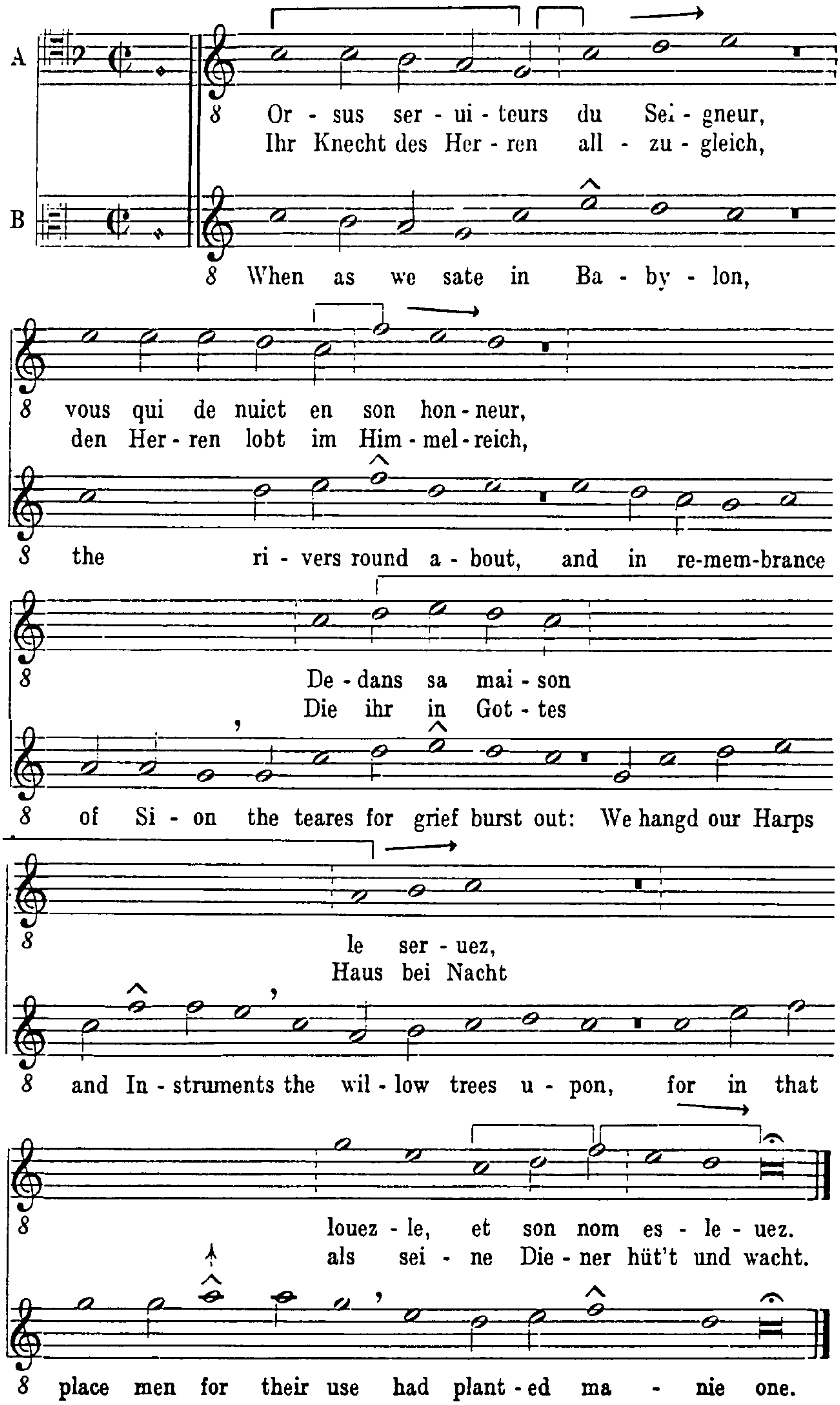

Fig. 5. (A) Psalm 134, 1551, French; (B) Psalm 137, 1555, English.

II. (a) Psalm 134 (1551, French), "Orsus serviteurs du Seigneur";[3] (b) Psalm 137, contrafact of (a) (1555, English), "When we sate in Babylon" (Fig. 5): (a) CP = 0.1875; (b) CP = 0.1607
a - b = 0.0268

III. Here the method is tested in two variants in *different languages:*[4] (a) "Es steht ein Lind in jenem Thal" (German); (b) "Šiel vojacek na vojm sám" (Slovakian) (Fig. 6).
(a) CP = 0.1666; (b) CP = 0.29032
b - a = 0.12372

Fig. 6. (A) "Es steht ein Lind in jenem Tal" (German);
(B) "Šiel vojacek na vojm sám (Slovakian).

3 *Ibid.*, p. 115.
4 *Ibid.*, p. 404.

Here the difference amounts to more than one per cent of the number of constituent tones, but it is still not decisive. In spite of the difference, some characteristic intervals are common to both pieces, as is the Lydian mode.

The last group of our comparisons is of twelve variants of the English ballad "Lady Isabel" and its contrafacts "The Outlandish Knight," "Tell-tale Polly," "May Collean," "The Seven King's Daughters," chosen at random by an aleatoric procedure. The notation of the variants used follows the Bronson's fine edition of the Child Ballads.[5] In order to remain completely unbiased, I have disregarded the editor's remarks, which are otherwise valuable and most interesting for the folklorist. The variants are:

1. no. 1. "[The Outlandish Knight]," CP = 0.1440 (Fig. 7).
2. no. 4. "[The Outlandish Knight]," CP = 0.24117 (Fig. 8).
3. no. 3. "[May Collean]," CP = 0.1335 (Fig. 9).
4. no. 2. "The Seven King's Daughters," CP = 0.25632 (Fig. 10).
5. no. 7."[The Outlandish Knight]," CP = 0.20086 (Fig. 11).
6. no. 11. "[The Outlandish Knight]," CP = 0.20588 (Fig. 12).
7. no. 25. "[Tell-tale Polly]," CP = 0.13034 (Fig. 13).
8. no. 43. "[Lady Isabel and the Elf Knight]," CP = 0.215346 (Fig. 14).
9. no. 49. "[Lady Isabel and the Elf Knight]," CP = 0.1090 (Fig. 15).
10. no. 52b. "[The Outlandish Knight]," CP = 0.151368 (Fig. 16).
11. no. 35. "The False-Hearted Knight," CP = 0.24163 (Fig. 17).
12. no. 38. "[The Outlandish Knight]," CP = 0.16935 (Fig. 18).

Fig. 7. Bronson no. 1. [The Outlandish Knight]

5 Bertrand Harris Bronson, *The Traditional Tunes of the Child Ballads* (Princeton: Princeton University Press, 1959), vol. I, pp. 42, 43, 44, 45, 50, 54, 56, 58, 60.

Fig. 8. Bronson no. 4. [The Outlandish Knight]

Fig. 9. Bronson no. 3. [May Collean]

Fig. 10. Bronson no. 2. "The Seven King's Daughters"

Fig. 11. Bronson no. 7. [The Outlandish Knight]

Fig. 12. Bronson no. 11. [The Outlandish Knight]

Fig. 13. Bronson no. 25. [Tell-tale Polly]

Fig. 14. Bronson no. 43. [Lady Isabel and the Elf Knight]

Fig. 15. Bronson no. 49. [Lady Isabel and the Elf Knight]

Fig. 16. Bronson no. 52b. [The Outlandish Knight]

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